MATRIX EXPERIMENTS

USING

ORTHOGONAL ARRAYS

MATRIX EXPERIMENTS USING ORTHOGONAL ARRAY

- DESCRIPTION OF 'CVD' PROCESS UNDER STUDY
- MATRIX EXPERIMENT AND CONCEPT OF ORTHOGONALITY
- ANALYSIS OF DATA OBTAINED FROM MATRIX EXPT.
- ADDITIVE MODEL
- ANALYSIS OF VARIANCE (ANOVA) AND ANALOGY BETWEEN ANOVA AND FOURIER ANALYSIS
- USE OF ADDITIVE MODEL FOR PREDICTION AND DIAGNOSIS

CVD PROCESS FOR POLYSILICON DEPOSITION IN VLSI TECHNOLOGY

- **CONTROL PARAMETERS (FACTORS)**
  - **TEMPERATURE (A), PRESSURE (B),**

    SETTLING TIME (C) AND CLEANING METHOD (D)
- **EACH PARAMETER HAS 3 SETTINGS (LEVELS) ONE OF WHICH IS THE STARTING LEVEL**
- **AIM IS TO MINIMISE THE SURFACE DEFECTS**
- **MATRIX EXPERIMENTS ARE PERFORMED SO AS TO OBTAIN OPTIMUM LEVELS OF FACTORS**

MATRIX EXPERIMENTS USING L9 ARRAY

- ONLY 9 EXPERIMENTS
  - OUT OF A TOTAL OF $3^4 = 81$
- FOUR CONTROL FACTORS ASSIGNED TO THE FOUR COLUMNS OF L9
- EACH ROW REPRESENTS AN EXPERIMENT HAVING A COMBINATION OF CONTROL FACTOR LEVELS

e.g. ROW #3 (EXPT. #3) --> A1 B2 C3 D3

and ROW #8 (EXPT. #8) --> A3 B2 C1 D3

L9 ORTHOGONAL ARRAY

EXPT. NO.	1 A	2 B	3 C	4 D
1	A1	B1	C1	D1
2	A1	B2	C2	D2
3	A1	B3	C3	D3
4	A2	B1	C2	D3
5	A2	B2	C3	D1
6	A2	B3	C1	D2
7	A3	B1	C3	D2
8	A3	B2	C1	D3
9	A3	B3	C2	D1

L9 ORTHOGONAL ARRAY

- **COLUMNS ARE ORTHOGONAL – IN THE SENSE THAT**
  " FOR ANY PAIR OF COLUMNS, ALL COMBINATIONS OF FACTOR LEVELS OCCUR AN EQUAL NUMBER OF TIMES "
- **COLUMNS ARE BALANCED,**
  WHICH ALSO IMPLIES ORTHOGONALITY
- **SIX PAIRS OF COLUMNS ($^4C_2$)**

? CAN YOU VERIFY ORTHOGONALITY FOR A FEW PAIRS ¿

L9 ORTHOGONAL ARRAY

EACH COMBINATION APPEARS only ONCE

EXPT. NO.	1 . A .	2 . B .	3 . C .	4 . D .
1		B1	C1	
2		B2	C2	
3		B3	C3	
4		B1	C2	
5		B2	C3	
6		B3	C1	
7		B1	C3	
8		B2	C1	
9		B3	C2	

L9 ORTHOGONAL ARRAY

with MEASURED SN-RATIO

EXPT. NO.	1 . A .	2 . B .	3 . C .	4 . D .	SN-RATIO η (in dB)
1	A1	B1	C1	D1	η1
2	A1	B2	C2	D2	η2
3	A1	B3	C3	D3	η3
4	A2	B1	C2	D3	η4
5	A2	B2	C3	D1	η5
6	A2	B3	C1	D2	η6
7	A3	B1	C3	D2	η7
8	A3	B2	C1	D3	η8
9	A3	B3	C2	D1	η9

FACTOR EFFECTS

- **EFFECT OF A FACTOR LEVEL IS DEFINED AS " THE DEVIATION IT CAUSES FROM OVERALL MEAN , m "**
- **FACTOR EFFECT OF TEMPERATURE LEVEL 3 :**
  - **A3 OCCURS IN EXPTS. 7, 8 AND 9**
  - **mA3 = 1/3 * (η 7 + η 8 + η 9)**
  - **FACTOR EFFECT OF A3 = a3 = mA3 - m**
- **REPEAT FOR ALL FACTORS AND ALL LEVELS**
  - **mA1 , mA2 , mB1 , ... , mD2 , mD3**

TABULAR AND GRAPHICAL REPRESENTATION OF FACTOR EFFECTS

- NUMERICAL VALUES ARE GIVEN IN A TABULAR FORM

or

- GRAPHICAL REPRESENTATION IS CONVENIENT FOR DRAWING QUALITATIVE INFERENCES AND CHOOSING THE OPTIMUM LEVELS OF FACTORS

L9 ORTHOGONAL ARRAY AND EXPERIMENTER'S LOG

TEMPERATURE COLUMN VARIABLE SHOWS HOW TO ENTER THE EXPT. LOG

EXPT. NO.	TEMPERATURE A	PRESSURE B	SILANE C	CLEANING METHOD D
1	T0-25--> 1	1	1	1
2	T0-25--> 1	2	2	2
3	T0-25--> 1	3	3	3
4	T0-----> 2	1	2	3
5	T0-----> 2	2	3	1
6	T0-----> 2	3	1	2
7	T0+25--> 3	P0-200--> 1	S0------> 3	CM2-----> 2
8	T0+25--> 3	P0------> 2	S0-100--> 1	CM3-----> 2
9	T0+25--> 3	3	2	1

EXPT #7 AND #8 SHOW THE EXPERIMENTER'S LOG

SELECTING OPTIMUM FACTOR LEVELS

- **PRIMARY GOAL** – TO MINIMISE DEFECTS
  – TO FIND OPTIMUM LEVELS
- **BEST LEVELS**
  - FOR TEMPERATURE → A1 → TO - 25
  - FOR PRESSURE → B1 → PO - 200
  - FOR SETTLING TIME → C2 → to + 8
  - FOR CLEANING METHOD → D2 → CM2
    or → D3 → CM3
- **OPTIMUM SETTINGS** → MINIMUM DEFECTS
- **A1 B1 C2 D2 or A1 B1 C2 D3 → how few ?**

ADDITIVE MODEL

- **AN ADDITIVE MODEL (ALSO CALLED SUPERPOSITION MODEL)**
  - **RELATIONSHIP BETWEEN RESPONSE VARIABLE AND CONTROL FACTORS**
  - **EFFECTS OF CONTROL FACTOR LEVELS IS ESTIMATED SEPARATELY**
  - **PREDICTION BY SIMPLY ADDING INDIVIDUAL FACTOR EFFECTS**
- **THE ADDITIVITY IS INFLUENCED BY THE CHOICE OF**
  - **QUALITY CHARACTERISTICS, S/N RATIO, CONTROL FACTORS / LEVELS**
- **BENEFITS OF ACHIEVING ADDITIVITY :**
  - **PREDICTIONS VALID FOR LAB AND MANUFACTURING CONDITIONS**

ADDITIVE MODEL (CONTD.)

- **VARIABLES SEPARABLE MODEL**
- **PROVIDES JUSTIFICATION FOR CALCULATING FACTOR EFFECTS AS AVERAGES OF APPROPRIATE EXPTS. (AS WAS DONE EARLIER)**
- **ADDITIVE MODEL AS AN APPROXIMATION**

$$\eta = \mu + a_i + b_j + c_k + d_l + \ldots + \epsilon$$

- **DEVIATION CAUSED BY LEVEL A_i FROM μ IS GIVEN BY a_i, BY B_j FROM μ AS b_j ETC.**
- **ϵ IS THE ERROR DUE TO ADDITIVE APPROXIMATION AS WELL AS ANY ERROR IN MEASUREMENT.**

ADDITIVE MODEL (contd.)

- **INDIVIDUAL FACTOR EFFECTS CAN BE LINEAR, QUADRATIC OR OF HIGHER ORDER**
- **CROSS - PRODUCT TERMS INVOLVING TWO OR MORE FACTORS ARE NOT ALLOWED**
- **SUM OF DEVIATIONS CAUSED BY 3 LEVELS OF EACH FACTOR IS ZERO**

  e.g. FOR FACTOR A → $a_1 + a_2 + a_3 = 0$

- **THE MEAN OF FACTOR EFFECTS OF 'ALL' FACTORS IS EQUAL TO 'μ'**

L8 ORTHOGONAL ARRAY

EXPT. NO.	1 A	2 B	3 C	4 D	5 E	6 F	7 G
1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	2
3	1	2	2	1	1	2	2
4	1	2	2	2	2	1	1
5	2	1	2	1	2	1	2
6	2	1	2	2	1	2	1
7	2	2	1	1	2	2	1
8	2	2	1	2	1	1	2

L9 ORTHOGONAL ARRAY

EXPT. NO.	1 A	2 B	3 C	4 D
1	A1	B1	C1	D1
2	A1	B2	C2	D2
3	A1	B3	C3	D3
4	A2	B1	C2	D3
5	A2	B2	C3	D1
6	A2	B3	C1	D2
7	A3	B1	C3	D2
8	A3	B2	C1	D3
9	A3	B3	C2	D1

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L18 ORTHOGONAL ARRAY

EXPT. NO.	1 A	2 B	3 C	4 D	5 E	6 F	7 G	8 H
1	1	1	1	1	1	1	1	1
2	1	1	2	2	2	2	2	2
3	1	1	3	3	3	3	3	3
4	1	2	1	1	2	2	3	3
5	1	2	2	2	3	3	1	1
6	1	2	3	3	1	1	2	2
7	1	3	1	2	1	3	2	3
8	1	3	2	3	2	1	3	1
9	1	3	3	1	3	2	1	2
10	2	1	1	3	3	2	2	1
11	2	1	2	1	1	3	3	2
12	2	1	3	2	2	1	1	3
13	2	2	1	2	2	1	3	2
14	2	2	2	3	3	2	1	3
15	2	2	3	1	1	3	2	1
16	2	3	1	3	3	3	1	2
17	2	3	2	1	1	1	2	3
18	2	3	3	2	2	2	3	1

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ACHIEVING ADDITIVITY

- RELATIONSHIP BETWEEN CONTROL FACTORS AND PERFORMANCE CHARACTERISTIC MUST BE ADDITIVE
- ADDITIVITY ENSURES STABILITY OF DESIGN UNDER LABORATORY , MANUFACTURING AND CUSTOMER USAGE

  > DESIGN IS ROBUST
- ADDITIVITY DEPENDS ON
  - CONTROL FACTOR SELECTION
  - QUALITY CHARACTERISTIC SELECTION
- LOG FORM OF OBJECTIVE FUNCTION

  ➔ IMPROVES ADDITIVITY

ESTIMATION OF FACTOR EFFECTS
USING ADDITIVE MODEL

- EFFECT OF SETTING TEMPERATURE AT LEVEL 3

  $mA3 = 1/3 * (\eta 7 + \eta 8 + \eta 9) = m + a3$
- EFFECT OF SETTING CLEANING METHOD AT LEVEL 3

  $mD3 = 1/3 * (\eta 3 + \eta 5 + \eta 9) = m + d3$

  etc. etc.

ANALYSIS OF VARIANCE (ANOVA)

- **RELATIVE MAGNITUDE OF FACTOR EFFECTS → 'QUALITATIVELY' FROM TABLES or GRAPHS**
- **ANOVA GIVES A 'QUANTITATIVE' MEASURE FOR THE RELATIVE MAGNITUDES OF DIFFERENT FACTOR EFFECTS**
- **ANOVA IS NEEDED FOR ESTIMATING THE**
  - **→ERROR VARIANCE FOR THE FACTOR EFFECTS and**
  - **→VARIANCE OF THE PREDICTION ERROR**

ANOVA - SUM OF SQUARES

- GRAND TOTAL SUM OF SQUARES $G_T = \sum_{i=1}^{9} \eta_i^2$
- SUM OF SQUARES DUE TO MEAN $S_M = 9 * m^2$
- TOTAL SUM OF SQUARES = $G_T - S_M$

  also $= \sum_{i=1}^{9} \eta_i^2 - 9 * m^2$

ANOVA - SUM OF SQUARES DUE TO FACTORS

- **SUM OF SQUARES DUE TO FACTOR 'A'**

  $SSQ\_A = 3 (a1^2 + a2^2 + a3^2)$

- **SUM OF SQUARES DUE TO FACTOR 'B'**

  $SSQ\_B = 3 (b1^2 + b2^2 + b3^2)$

- **SUM OF SQUARES DUE TO FACTOR 'C'**

  $SSQ\_C = 3 (C1^2 + C2^2 + C3^2)$

- **SUM OF SQUARES DUE TO FACTOR 'D'**

  $SSQ\_D = 3 (d1^2 + d2^2 + d3^2)$

DEGREES OF FREEDOM and VARIANCE RATIO 'F'

- **NUMBER OF INDEPENDENT PARAMETERS OR LEVELS ASSOCIATED WITH A**
  - **MATRIX EXPERIMENT**
  - **FACTOR**

  or
  - **SUM OF SQUARES**
- **TOTAL DEGREES OF FREEDOM DECIDE WHICH ORTHOGONAL ARRAY IS SUITABLE**
- **DETERMINE → SUM OF SQUARES DUE TO ERROR**
- **DETERMINE THE VARIANCE RATIO 'F' FOR A FACTOR**

$$F = \frac{\text{MEAN OF SUM OF SQUARES DUE TO A FACTOR}}{\text{MEAN SQUARE ERROR}}$$

ERROR VARIANCE AND POOLING OF ERRORS

- ERROR VARIANCE –

$$\sigma_e^2 = \frac{\text{SUM OF SQUARES DUE TO ERROR}}{\text{DEGREES OF FREEDOM FOR ERROR}}$$

- POOLING OF ERRORS
  - WHEN DEGREES OF FREEDOM FOR ERROR ARE ZERO

  or
  - WHEN ONE OR MORE FACTORS HAVE A SMALL EFFECT ON THE OBJECTIVE FUNCTION

CONFIDENCE INTERVALS FOR FACTOR EFFECTS

- ERROR VARIANCE –

$$\sigma_e^2 = \frac{\text{SUM OF SQUARES DUE TO ERROR}}{\text{DEGREES OF FREEDOM FOR ERROR}}$$

- FOR 95% CONFIDENCE INTERVAL –
  - CHOOSE TWO STANDARD DEVIATIONS FOR ERROR AS THE CONFIDENCE INTERVAL
  - CONFIDENCE INTERVAL = ± 2 [SQRT (ERROR VARIANCE)] dB

VARIANCE RATIO 'F' and INTERPRETATION OF ANOVA TABLES

- **VARIANCE RATIO 'F' –**

$$F = \frac{\text{MEAN SUM OF SQUARES DUE TO A FACTOR}}{\text{ERROR MEAN SQUARE}}$$

- **F < 1 – FACTOR EFFECT IS SMALLER THAN THE ERROR OF THE ADDITIVITY MODEL**
- **F > 2 – FACTOR EFFECT NOT QUITE SMALL**
- **F > 4 – FACTOR EFFECT IS QUITE LARGE**

$$\%\ \text{CONTRIBUTION} = \frac{\text{SUM OF SQUARES DUE TO A FACTOR}}{\text{TOTAL SUM OF SQUARES}}$$

PREDICTION OF IMPROVEMENT

- OPTIMUM QUALITY FOR , say , A1 B1 (C2 D2)

➔ $\eta_{OPT} = m + a_1 + b_1$

➔ $\eta_{OPT} = m + (mA1 - m) + (mB1 - m)$ **NO CHANGE**

- IMPROVEMENT OVER NOMINAL (A2 B2 C2 D2)
- $\eta_{OPT} - \eta_{NOM} = (a_1 - a_2) + (b_1 - b_2)$

CONFIRMATION EXPERIMENTS

- **TO VERIFY THAT THE PREDICTION IS CORRECT**

- **TO VERIFY WHETHER ADDITIVE MODEL IS SATISFIED**

– IF YES > ADOPT THE ' NEW ' SETTINGS

– IF NO > FAILURE OF ADDITIVE MODEL

and

INDICATION OF STRONG INTERACTION AMONGST TWO OR MORE FACTORS

therefore

PLAN FUTURE ACTION

INTERACTION AMONGST FACTORS

- **DESIRABLE – NO INTERACTIONS**
  – SATISFY ADDITIVITY

- **SYNERGISTIC INTERACTION – LEADS TO SOME ERROR**
  – BUT IT IS ACCEPTABLE

- **ANTI-SYNERGISTIC INTERACTION –**

> FAILURE OF ADDITIVE MODEL

> SELECT APPROPRIATE S / N RATIO

> PERFORM FRESH MATRIX EXPERIMENTS TO STUDY SPECIFIC INTERACTIONS

EXAMPLES OF ORTHOGONAL ARRAYS

L8 ORTHOGONAL ARRAY

EXPT. NO.	1 A	2 B	3 C	4 D	5 E	6 F	7 G
1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	2
3	1	2	2	1	1	2	2
4	1	2	2	2	2	1	1
5	2	1	2	1	2	1	2
6	2	1	2	2	1	2	1
7	2	2	1	1	2	2	1
8	2	2	1	2	1	1	2

L9 ORTHOGONAL ARRAY

EXPT. NO.	1 A	2 B	3 C	4 D
#1	1	1	1	1
#2	1	2	2	2
#3	1	3	3	3
#4	2	1	2	3
#5	2	2	3	1
#6	2	3	1	2
#7	3	1	3	2
#8	3	2	1	3
#9	3	3	2	1

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L18 ORTHOGONAL ARRAY

EXPT. NO.	1 A	2 B	3 C	4 D	5 E	6 F	7 G	8 H
1	1	1	1	1	1	1	1	1
2	1	1	2	2	2	2	2	2
3	1	1	3	3	3	3	3	3
4	1	2	1	1	2	2	3	3
5	1	2	2	2	3	3	1	1
6	1	2	3	3	1	1	2	2
7	1	3	1	2	1	3	2	3
8	1	3	2	3	2	1	3	1
9	1	3	3	1	3	2	1	2
10	2	1	1	3	3	2	2	1
11	2	1	2	1	1	3	3	2
12	2	1	3	2	2	1	1	3
13	2	2	1	2	2	1	3	2
14	2	2	2	3	3	2	1	3
15	2	2	3	1	1	3	2	1
16	2	3	1	3	3	3	1	2
17	2	3	2	1	1	1	2	3
18	2	3	3	2	2	2	3	1

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L8 ORTHOGONAL ARRAY AND EXPERIMENTER'S LOG
FOR DRILLING HOLE IN A CAST ALLOY WHEEL

EXPT. NO.	SPEED A	FEED RATE B	DRILL LENGTH C	COOLANT D	FIXTURE E	RESPONSE S / N
1	1500	15	4	BLUE	OLD	42.18
2	1500	15	7	WHITE	NEW	53.96
3	1500	18	4	WHITE	NEW	48.48
4	1500	18	7	BLUE	OLD	48.98
5	2000	15	4	BLUE	NEW	44.20
6	2000	15	7	WHITE	OLD	42.19
7	2000	18	4	WHITE	OLD	41.58
8	2000	18	7	BLUE	NEW	56.40

L18 ORTHOGONAL ARRAY AND EXPERIMENTER'S LOG
COMPUTER TUNING for HIGH PERFORMANCE

EXPT. NO.	1 F	2 B	3 C	4 D	5 E	6 A	7 G	8 H
1	2	a	4	1 / 5	0	4 & 1	400	a
2	2	a	3	1 / 4	3	4 & 2	500	b
3	2	a	3.5	1 / 3	8	4 & 2	600	c
.								
.								
.								
.								
.								
16	0	c	4	1 / 3	3	4 & 2	400	b
17	0	c	3	1 / 5	8	4 & 2	500	c
18	0	c	3.5	1 / 4	0	4 & 1	600	a

ROLE OF ORTHOGONAL ARRAYS

- OPTIMIZATION BY STUDYING ONE FACTOR AT A TIME DOES NOT GIVE ANY IDEA ABOUT INTERACTIONS
- ABSENCE OF ADDITIVITY IS POINTED OUT BY THE FAILURE OF VERIFICATION EXPERIMENT WHICH IN TURN POINTS TO LARGE INTERACTIONS
- DEVIATION FROM ADDITIVITY QUANTIFIES LARGENESS OF INTERACTIONS RELATED TO MAIN EFFECTS
- RECHECK THE CHOICE OF CONTROL FACTORS AND QUALITY CHARACTERISTICS

§ IF THE VERIFICATION EXPERIMENT SUCCEEDS, THE PRODUCT / PROCESS IS PROVED TO BE ROBUST (AGAINST NOISE FACTORS)

Thank You