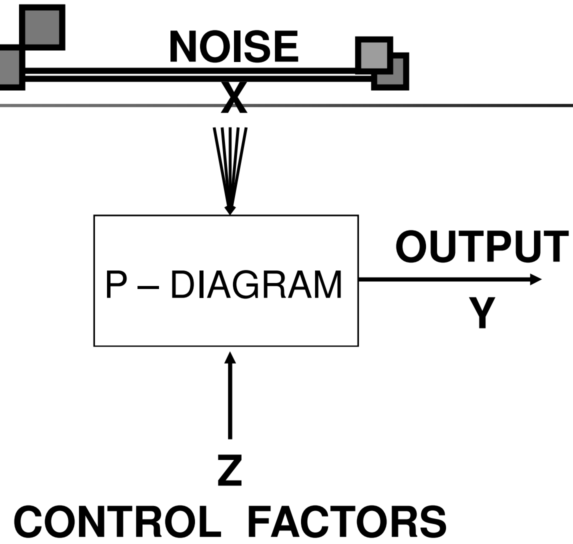
TAGUCHI METHOD for DYNAMIC PROBLEMS

Dr. P. R. Apte
IIT Bombay

SIGNAL - TO - NOISE RATIO



P – Diagram for STATIC Problems

QUADRATIC LOSS FUNCTION

- $Q = K' [(\mu - \mu_o)^2 + \sigma^2]$
- Quadratic Loss Function
  - Ideal measure for quality of products as it is shipped by the supplier to the customer
- Minimizing Quadratic Loss Function
  - leads to quality improvement
- Signal-to-Noise ratios are

  Log form of Quadratic Loss Function

QUALITY LOSS AFTER ADJUSTMENT

- First Variance is reduced → ("σ^2 term" in 'Q')
- Then Mean is brought on Target without disturbing the Variance (σ^2) by a Scaling Factor
- Hence the correct Quality Loss measure is Q_a

  $Q_a = K\,\sigma^2$ for ($\mu=\mu o$) and NOT

  $Q = K\,[(\mu - \mu_o)^2 + \sigma^2]$
- Hence always use Q_a and not Q for any estimation of quality

RELATIONSHIP BETWEEN S/N RATIO AND Q_a

- $Q = K' [(\mu - \mu_o)^2 + \sigma^2]$
- If mean is μ and is to be 'adjusted' to μ_o ,
  - Adjust Thickness to 'μ_o / μ' times the earlier values
- $Q_a = K' [(\mu . \{\mu_o / \mu\} - \mu_o)^2 + (\sigma . \{\mu_o / \mu\})^2]$
- $Q_a = K' [(\mu_o - \mu_o)^2 + (\sigma . \{\mu_o / \mu\})^2]$
- $Q_a = K' [(\sigma . \{\mu_o / \mu\})^2]$

RELATIONSHIP BETWEEN S/N RATIO AND Q_a

- Q_a = K' [(σ . {μ_o / μ}) ^{2}]
- Q_a = K'. {μ_o} ^{2} [(σ / μ) ^{2}]
- Q_a = K (σ / μ) ^{2}
  - where K, new constant = K'. {μ_o} ^{2}
- Q_a minimizing is equivalent to maximizing {1/Q_a}
- S/N Ratio, η = Log Form of {1/Q_a}

$$Q_{a'} = \eta = 10 \text{ Log}_{10} [\mu^2 / \sigma^2]$$

TAGUCHI'S TWO-STEP PROCEDURE

- What you need to do?
  1. Maximize η = 10 Log_{10} [μ ^{2} / σ ^{2}]
  2. Adjust Mean on Target without disturbing the Variance by a Scaling Factor (eg. deposition time)
- Any target can be adjusted without having to re-optimize
  - → Unconstrained optimization
  - → Development of sub-systems to take place in parallel
  - → R & D time reduced

ROBUST DESIGN METHODOLOGY

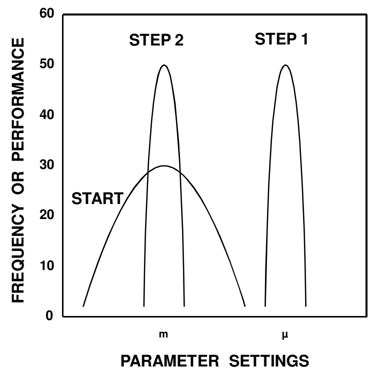
2 - STEP OPTIMIZATION

STEP 1 :

REDUCE VARIATION IRRESPECTIVE OF TARGET VALUE

STEP 2 :

ADJUST PERFORMANCE ON TARGET LEAVING VARIATION UNDISTURBED



IDENTIFICATION OF SCALING FACTOR

- DOES EVERY CONTROL FACTOR HAVE EFFECT ON 'η' AND 'μ' ?
- FACTORS THAT HAVE SIGNIFICANT EFFECT ON 'η' ---> MAXIMIZE 'η'
- FACTORS THAT HAVE SIGNIFICANT EFFECT ON 'μ' BUT NO EFFECT ON 'η'

  ---> SCALING FACTOR
- FACTORS THAT NEITHER EFFECT 'η' NOR 'μ'

  ---> NEUTRAL FACTORS

  (OPERATIONAL EASE, COST ETC.)

S/N RATIOS FOR STATIC PROBLEMS

- **TYPES OF STATIC PROBLEMS**
  - SMALLER - THE - BETTER TYPE
  - NOMINAL - THE - BEST TYPE
  - LARGER - THE - BETTER TYPE
- **MORE TYPES**
  - SIGNED - TARGET TYPE
  - FRACTION DEFECTIVE TYPE
  - ORDERED CATEGORIES TYPE
  - CURVE OR VECTOR RESPONSE TYPE

KEY ACTIVITY OF ROBUST DESIGN

IS TO :

- **DETERMINE CORRECT S/N RATIO SO AS TO ACHIEVE ADDITIVITY**
- **DETERMINE PROPER ADJUSTMENT FACTORS TO BRING MEAN ON TARGET WITHOUT DISTURBING VARIANCE**

SMALLER - THE - BETTER TYPE PROBLEM

- QUALITY CHARACTERISTICS 0 TO ∞
  (CONTINUOUS AND NON-NEGATIVE)
- **' 0 ' MOST DESIRED**
- **NO SCALING OR ADJUSTMENT FACTOR AS TARGET IS ALWAYS ZERO**
- VARIANCE TENDS TO ZERO AS MEAN TENDS TO ZERO
- **MINIMIZE QUALITY LOSS WITHOUT ADJUSTMENT**
- **OR MAXIMIZE**

$$\eta = -10 \, \text{Log}_{10} \left[\frac{1}{n} \sum (Y_1^2 + Y_2^2 + \ldots + Y_n^2) \right]$$

NOMINAL - THE - BEST TYPE PROBLEM

- QUALITY CHARACTERISTICS 0 TO ∞
  (CONTINUOUS AND NON-NEGATIVE)
- MOST DESIRED OR TARGET VALUE IS NON-ZERO AND FINITE
- POSSIBLE TO FIND A SCALING OR ADJUSTMENT FACTOR
- WHEN MEAN IS ZERO, STANDARD DEVIATION IS ALSO ZERO
- **MAXIMIZE**

$$\eta = 10 \, \text{Log}_{10} \left[\mu^2 / \sigma^2 \right]$$

NOMINAL - THE - BEST TYPE PROBLEM

- **MAXIMIZE**

$$\eta = 10 \text{ Log}_{10} [\mu^2 / \sigma^2]$$

- **USE TWO - STEP PROCEDURE**

1. Minimize Variance $\{ \sigma^2 \}$
2. Adjust the mean 'μ' on to target 'μ_o'

LARGER - THE - BETTER TYPE PROBLEM

- QUALITY CHARACTERISTICS IS 0 TO ∞ (CONTINUOUS AND NON-NEGATIVE)
- NO SCALING OR ADJUSTMENT FACTOR
- CAN BE TRANSFORMED INTO SMALLER - THE - BETTER TYPE
- CONSIDER THE RECIPROCAL OF QUALITY CHARACTERISTICS
- MAXIMIZE

$$\eta = -10 \text{ Log}_{10} \left[\frac{1}{n} \sum (1/Y_1^2 + 1/Y_2^2 + ... + 1/Y_n^2) \right]$$

SIGNED - TARGET TYPE PROBLEM

- QUALITY CHARACTERISTICS CAN TAKE POSITIVE AND NEGATIVE VALUES
- OFTEN, TARGET VALUE = ZERO
  IF NOT, IT CAN BE MADE
  ZERO BY CHANGE OF REFERENCE
- WHEN MEAN ***IS*** ZERO,
  STANDARD DEVIATION IS ***NOT*** ZERO
- A SCALING OR ADJUSTMENT FACTOR EXISTS
- **MAXIMIZE**

$$\eta = - 10 \, \text{Log}_{10} \, [\, \sigma^{2} \,]$$

FRACTION DEFECTIVE TYPE PROBLEM

- QUALITY CHARACTERISTICS IS A FRACTION P (0 TO 1)
- TARGET VALUE = ZERO
- NO SCALING OR ADJUSTMENT FACTOR
- **MAXIMIZE**

$$\eta = - 10 \, \text{Log}_{10} \, [\, P / (1 - P) \,]$$

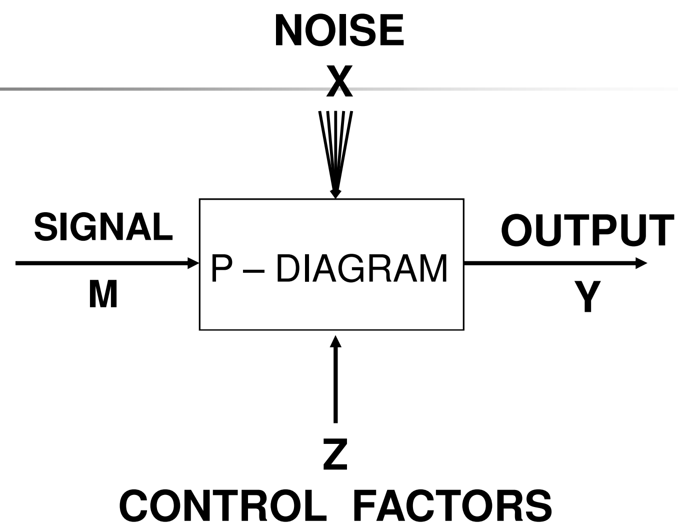
- OR MINIMIZE
  - NO. OF BAD PIECES TO PRODUCE ONE GOOD PIECE

ORDERED CATEGORICAL PROBLEM

- QUALITY CHARACTERISTICS TAKE ORDERED CATEGORICAL VALUES
  (C1 = WORSE C2 = NO CHANGE
  C3 = GOOD C4 = EXCELLENT)
- EXTREME CATEGORY C4 IS MOST DESIRED (TARGET VALUE)
- FORM CUMULATIVE CATEGORIES.
- TREAT EACH CATEGORY AS
- A FRACTION DEFECTIVE PROBLEM

CURVE OR VECTOR TYPE PROBLEM

- QUALITY CHARACTERISTICS IS A CURVE OR A VECTOR
- BROKEN INTO SEVERAL SCALAR PROBLEMS OF THE TYPE ALREADY DESCRIBED



P – Diagram for DYNAMIC Problems

S/N RATIOS FOR DYNAMIC PROBLEMS

- TYPES OF DYNAMIC PROBLEMS
  - CINTINUOUS - CONTINUOUS TYPE (C - C)
  - CONTINUOUS - DIGITAL TYPE (C - D)
  - DIGITAL - CONTINUOUS TYPE (D - C)
  - DIGITAL - DIGITAL TYPE (D - D)

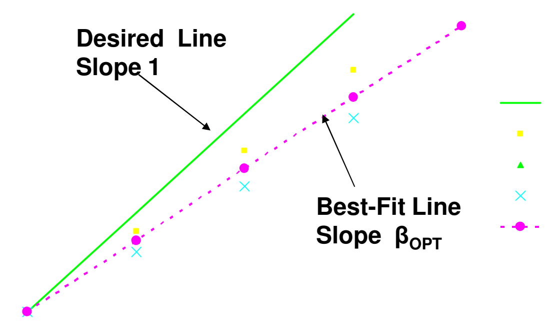
CONTINUOUS - CONTINUOUS TYPE PROBLEM

- BOTH SIGNAL FACTOR AND QUALITY CHARACTERISTICS TAKE POSITIVE OR NEGATIVE VALUES
- WHEN SIGNAL M = 0 , QUALITY CHARACTERISTIC = 0
- IDEAL FUNCTION y = M
- SCALING FACTOR EXISTS TO ADJUST SLOPE (PROPORTIONALITY CONSTANT) BETWEEN ' y ' AND ' M '

S/N RATIO FOR A C - C TYPE PROBLEM

- QUALITY CHARACTERISTIC ' y ', FOLLOWS THE SIGNAL FACTOR ' M '
- y = β M (usually β is taken as 1) → y = M

Dynamic Characteristics, Y = M



S/N RATIO FOR A C - C TYPE PROBLEM

- Quality Loss for each $Y_{ij} = K (Y_{ij} - M_i)$
- AVERAGE $$Q_Z = \frac{K}{m\,n} \sum_{I=1}^{m} \sum_{j=1}^{n} (Y_{ij} - M_i)^2$$
- QUALITY LOSS WITHOUT ADJUSTMENT HAS TWO COMPONENTS
  - LOSS DUE TO SLOPE NOT BEING EQUAL TO 1
  - LOSS DUE TO DEVIATION FROM LINARITY (VARIANCE)

REGRESSION OF ' Yij ' UPON Mi

- Let straight line fitting the ' Yij ' have a slope 'β'
- Quality Loss with respect to the straight line

$$Q_{Z'} = \frac{K}{m\,n} \sum_{I=1}^{m} \sum_{j=1}^{n} (Y_{ij} - \beta M_i)^2$$

BY METHOD OF LEAST SQUARES

- (Differentiate wrt β and equate to zero)

$$\beta_{OPT} \text{ (slope of best fit line)} = \frac{\sum_{I=1}^{m} \sum_{j=1}^{n} (Y_{ij} M_i)}{\sum_{I=1}^{m} \sum_{j=1}^{n} M_i^2}$$

- – BEST ADJUSTMENT
  - Multiply all ' **Yij** ' by $(1/\beta_{OPT})$
- SUCH THAT
  - Quality loss after adjustment

$$Q_{a'} = \frac{K}{m\,n} \sum_{I=1}^{m} \sum_{j=1}^{n} \left(\frac{Y_{ij}}{\beta_{OPT}} - \beta M_i \right)^2$$

[ADJUSTMENT FACTOR IS ($\mathbf{1}/\boldsymbol{\beta}_{\mathbf{OPT}}$)]

- Quality after adjustment, '$\mathbf{Q_a}$'

$$Q_{a'} = \frac{K}{m\,n} \sum_{I=1}^{m} \sum_{j=1}^{n} \left[\frac{(Y_{ij} - \beta M_i)^2}{\beta_{OPT}^{\ 2}} \right]$$

$$Q_{a'} = \frac{1}{\beta_{OPT}^{\ 2}} \boxed{\frac{K}{m\,n} \sum_{I=1}^{m} \sum_{j=1}^{n} (Y_{ij} - \beta M_i)^2}$$

$$Q_{a'} = \frac{\sigma_e^{\ 2}}{\beta_{OPT}^{\ 2}}$$

This is $\sigma_e^{\ 2}$

- Quality after adjustment, 'Q_a'

$$Q_{a'} = \frac{\sigma_e^2}{\beta_{OPT}^2}$$

- Minimize, 'Q_a' (**or maximize 1/ 'Q_a'**)

$$\eta = 10 \, \mathrm{Log}_{10} \left[\frac{\beta_{OPT}^2}{\sigma_e^2} \right]$$

(REDUCES NON-LINEARITY ALONG WITH REDUCTION IN SENSITIVITY TO NOISE FACTORS)

TWO STEP PROCEDURE to Optimize Dynamic Problems

- STEP 1

→ MAXIMIZE $$\eta = 10 \, \mathrm{Log}_{10} \left[\frac{\beta_{OPT}^2}{\sigma_e^2} \right]$$

- STEP 2

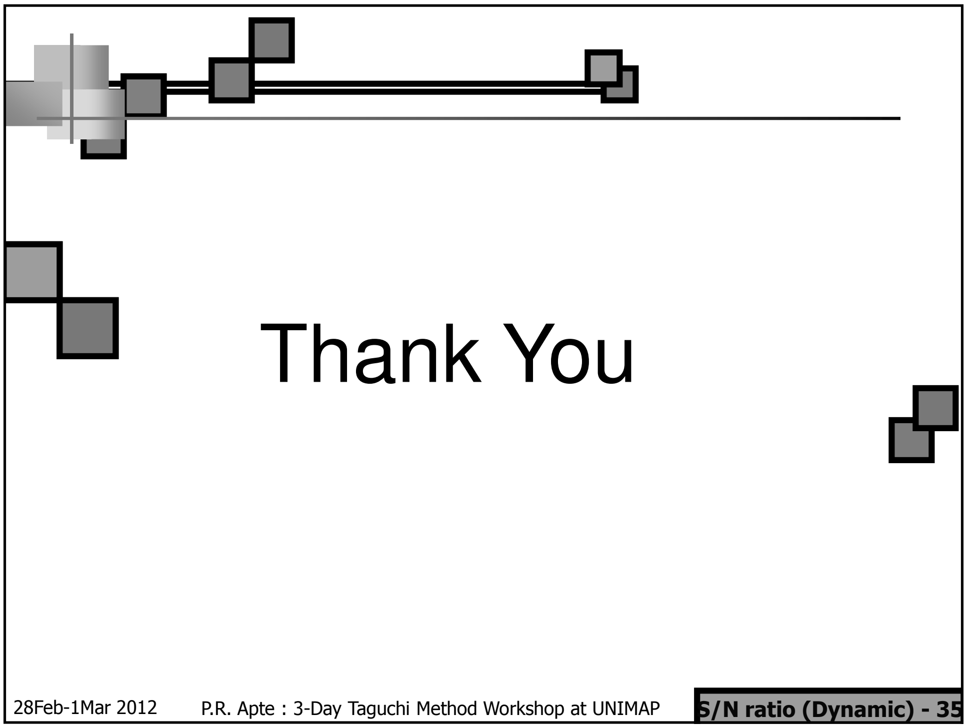
→ ADJUST SLOPE BY SUITABLE SCALING FACTOR

CONTINUOUS - DIGITAL TYPE (C - D)

- TEMPERATURE CONTROLLER
  - INPUT TEMPERATURE SETTING - CONTINUOUS
  - OUTPUT OF HEATING UNIT - ' ON ' OR ' OFF '
  - DIVIDE INTO TWO SEPARATE PROBLEMS
    ONE FOR ' ON ' FUNCTION
    OTHER FOR ' OFF ' FUNCTION
  - EACH ONE CONTINUOUS - CONTINUOUS TYPE OR
    NOMINAL - THE - BEST TYPE PROBLEM

DIGITAL - CONTINUOUS TYPE (D - C)

- DIGITAL TO ANALOG CONVERTER
  - CONVERSION TO ' 0 ' AND ' 1 '
  - DIVIDE INTO TWO SEPARATE PROBLEMS
    ONE FOR ' 0 ' FUNCTION
    OTHER FOR ' 1 ' FUNCTION
  - EACH ONE CONTINUOUS - CONTINUOUS TYPE OR
    NOMINAL - THE - BEST TYPE PROBLEM


Thank You
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P.R. Apte : 3-Day Taguchi Method Workshop at UNIMAP
S/N ratio (Dynamic) - 35